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A METHOD OF DETERMINING UNSTEADY AIR PERMEABILITY OF ROCK

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Abstract

It is important for utilization of underground space or slope disaster prevention to determine the water transport in the rock mass including unsaturated zones. To precisely predict the two-phase flow of air and water in rock pore spaces, both the hydraulic conductivity and gas permeability must be accurately identified. Most existing methods estimate air permeability under steady-state conditions. There are still few methods for evaluating air permeability under unsteady state conditions. This study proposes a method of identifying the ever-changing permeability with time and pressure using a linearized governing equation for air permeability in porous media. To validate the method, we compared the exact solution derived with the results of air permeability laboratory tests on granite and conglomerate in previous studies. After confirming that the exact solution well represented the results of previous experiments, the unsteady air permeability of the rock was estimated from the results of the same experiments using the proposed method. As a result, it was confirmed that not only can the air permeability be identified with the same accuracy as the steady-state method, but also the unsteady air permeability, which varies with pressure, can be identified.

Key words

Air peameability, unsteady, unsaturated flow, rock, geological disposal, laboratory test.

1 Introduction

It is important for geological disposal of nuclear waste and large-scale collapse of rock slopes due to heavy rain fall to understand the properties of unsaturated rocks because the mechanical properties of rocks change significantly with water content (e.g. Chigira, 2015; Osada, 2014). The authors have been studying changes in the strength and deformation properties of sedimentary rocks associated with water content (Togashi and Imano et al., 2021; Togashi and Kikumoto et al., 2021; Kotabe et al., 2024) and water transport in the excavation disturbed zone (Osada et al., 2019; Togashi et al., 2022). To precisely predict the two-phase flow of air and water in rock pore spaces, both the hydraulic conductivity and gas permeability must be accurately identified. Most existing methods estimate the air permeability from the differential pressure under steady-state conditions (e.g. Sakaguchi et al., 1992). There are still few methods for evaluating air permeability under unsteady state conditions.

This study proposes a method of identifying the ever-changing permeability with time and pressure using a linearized governing equation for air permeability in porous media. This method is based on an exact solution of the linear diffusion equation. To validate the method, we then compared the exact solution derived with the results of air permeability laboratory tests on granite and conglomerate in previous studies. After confirming that the exact solution well represented the results of previous experiments, the unsteady air permeability of the rock was estimated from the results of the same experiments using the proposed method.

2 Methods

2.1 Non-liner partial differential equation of unsteady gas flow

Below, the governing equation of unsteady gas flow is derived according to the method of Katz (1959). The ideal gas law is as follows.

$$\rho = \frac{M}{RT}P \tag{1}$$

where *r*, *M*, *P*, *R* and *T* are density, amount of substance, air pressure, gas constant and temperature, respectively. Darcy's law is given by the following equation, where the flow velocity is *u*.

$$u = -\frac{K}{\mu}\frac{\partial P}{\partial x} \tag{2}$$

Here, *K* and *m* are intrinsic air permeability coefficient and viscosity coefficient. The following is the continuous equation.

$$\frac{\partial \rho u}{\partial x} = -\lambda \frac{\partial \rho}{\partial t} \tag{3}$$

where *l* is porosity. Substituting Eq. (1) into Eq. (2) to eliminate P and substituting Eq. (3) yields:

$$\frac{\partial \rho}{\partial t} = \frac{K}{\mu\lambda\gamma}\frac{\partial}{\partial x}\left(\rho\frac{\partial \rho}{\partial x}\right) \tag{4}$$

Here, *M* / *RT* = *g* is set. The partial differential on the right side of this equation can be transformed as follows.

$$\frac{1}{2}\frac{\partial^2 \rho^2}{\partial x^2} = \frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial \rho^2}{\partial x}\right) = \frac{1}{2}\frac{\partial}{\partial x}\left(2\rho\frac{\partial \rho}{\partial x}\right) = \frac{\partial}{\partial x}\left(\rho\frac{\partial \rho}{\partial x}\right) \tag{5}$$

The one-dimensional air permeability phenomenon is expressed by the following governing equation for density.

$$\frac{\partial \rho}{\partial t} = \frac{K}{2\lambda\mu\gamma}\frac{\partial^2 \rho^2}{\partial x^2} \tag{6}$$

Here, if Eq. (1) is assigned to *r* in Eq. (6) with *M* / *RT* (= *g*) as a constant, the governing equation of the air permeability phenomenon for *P* is obtained and *g* disappears as follows.

$$\frac{\partial P}{\partial t} = \frac{K}{2\lambda\mu}\frac{\partial^2 P^2}{\partial x^2} \tag{7}$$

This equation is well known as the governing equation for the diffusion of porous media.

2.2 Simplification of the governing equation and exact solution for unsteady gas flow

As shown in Fig. 1, let the air pressures acting on x = 0 and x = l be P_0 and P_l, respectively. If the difference between P_0 and P_l is small, the pressure in the analysis area is small, so it is assumed to liner relationships $P_2 \simeq (P_0 + P_l)P$. This means that the difference between P_2 and $(P_0 + P_l)P$ is small around $P = P_0 + P_l$. Based on this assumption, the governing equation of unsteady gas flow, Eq. (7), becomes the following linear diffusion equation.

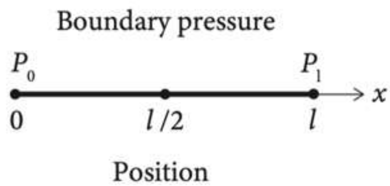

Figure 1. Problem setting of one dimensional gas flow.

$$\frac{\partial P}{\partial t} = G\frac{\partial^2 P}{\partial x^2} \tag{8}$$

It should be noted that $G = K(P_0 + P_l) / (2lm)$. By changing the variables as $h = x / 2(Gt)^{0.5}$, the infinitesimal increment of time, Dt and the square of the infinitesimal increment of coordinates, Dx^2, can be expressed as follows.

$$d\eta = -\frac{\eta}{2t}dt, \quad d\eta^2 = \frac{1}{4Gt}dx^2 \tag{9}$$

Substituting these small increments into Eq. (8) gives:

$$\frac{d^2P}{d\eta^2} + 2\eta\frac{dP}{d\eta} = 0 \tag{10}$$

Equation (10) is a second-order homogeneous differential equation of variable coefficients, which is solved under the following boundary conditions.

$$\begin{aligned} &x = 0 \quad (\eta = 0), \qquad P = P_0 \\ &x = l \quad (\eta = \alpha = \frac{l}{2\sqrt{Gt}}), \qquad P = P_l \end{aligned} \tag{11}$$

When $dh = D$, Eq. (10) becomes a first-order linear differential equation as follows.

$$\frac{dD}{d\eta} + 2\eta D = 0 \tag{12}$$

When this equation is separated into variables, integrated on both sides and solved to obtain the equation for P, it becomes as follows.

$$P = c \int_0^{\eta} e^{-\eta^2} d\eta + F \tag{13}$$

where c and F are arbitrary constants of integration. From the boundary condition, $P(h = 0) = F = P_0$ and $c = (P_l - P_0) / \int \exp(-h^2)\, dh$. The unsteady exact solution can be obtained by the following equation using the error function $\mathrm{erf}(x) = (2 / \pi^{0.5}) \int \exp(-t^2)\, dt$. Note that the interval of the integral sign in the sentence is $[0, x]$.

$$P = (P_l - P_0) \frac{\mathrm{erf}(\eta)}{\mathrm{erf}(\alpha)} + P_0 \tag{14}$$

2.3 Air permeability coefficient determination detection using un-steady exact solution

Here, we show how to specify the intrinsic permeability coefficient and unsaturated hydraulic conductivity using the exact solution of unsteady gas flow derived in the previous section. As shown in Fig. 1, a method for identifying the intrinsic air permeability coefficient K is shown by measuring the air pressure at the pressure boundary ($x = 0, L$) and the air pressure at one point inside the rock specimen ($x = L/2$). If the error function in the exact solution is expressed by a simple mathematical formula, the intrinsic air permeability coefficient K can be specified. The Taylor expansion of the error function is (Abramowitz et al. 1965):

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)} \tag{15}$$

By rearranging Eq. (14) and setting the ratio of the error functions to the symbol R, the following is obtained.

$$\frac{\mathrm{erf}(\eta)}{\mathrm{erf}(\alpha)} = \frac{P - P_0}{P_l - P_0} = R \tag{16}$$

At the position inside the specimen $x = l / 2$, $h = 0.5\, l / (2\, (Gt)^{0.5}) = 0.5\, a$. Therefore, using the Taylor expansion of the error function up to the first-order term, the above equation becomes as follows.

$$R = \frac{\mathrm{erf}(\eta)}{\mathrm{erf}(\alpha)} = \frac{\eta - \frac{\eta^3}{3}}{\alpha - \frac{\alpha^3}{3}} = \frac{\frac{1}{2}\alpha - \frac{\alpha^3}{24}}{\alpha - \frac{\alpha^3}{3}} = \frac{12 - \alpha^2}{24 - 8\alpha^2} \tag{17}$$

By transforming this into an expression, expressing a^2 by R, and substituting $a = l / 2(Gt)^{0.5}$ following is obtained.

$$\alpha^2 = \frac{24R - 12}{8R - 1} = \frac{l^2}{4Gt} \tag{19}$$

Here, $G = K(P_0 + P_l) / 2lm$, so G is represented by R as follows.

$$G = \frac{K(P_0 + P_l)}{2\lambda\mu} = \frac{l^2}{4t}\frac{8R-1}{24R-12} \tag{20}$$

From the above, the following equation for finding K is obtained.

$$\begin{aligned} K &= \frac{2\lambda\mu}{P_0 + P_l}\frac{l^2}{4t}\frac{8R-1}{24R-12} \\ &= \frac{\lambda\mu}{P_0 + P_l}\frac{l^2}{2t}\frac{8\frac{P-P_0}{P_l-P_0}-1}{24\frac{P-P_0}{P_l-P_0}-12} \end{aligned} \tag{21}$$

P_0, P, and P_l are barometric pressure increments measured at the start, midpoint, and end points, respectively. In this way, if there are measurement data at three points, the intrinsic air permeability coefficient K can be explicitly obtained. In this equation, the back of the right side is the pressure ratio function, $f(R) = (8R-1) / (24R-12)$. The outline of this function is as shown in Fig. 2. $f(R)$ diverges at $R = 0.5$. On the other hand, if R deviates from 0.5, it shows a value close to constant ($f(R) \approx 1$). Based on these properties, in the next section, we verify the method for specifying Eq. (21). The conversion between the intrinsic permeability coefficient and the unsaturated permeability coefficient is also described. According to Taylor (1943), the relationship between the intrinsic permeability coefficient K [m^2] and the unsaturated hydraulic conductivity k_w [m/s] is expressed by the following equation.

$$k_w = K\frac{\rho_w g}{\mu_w} \tag{22}$$

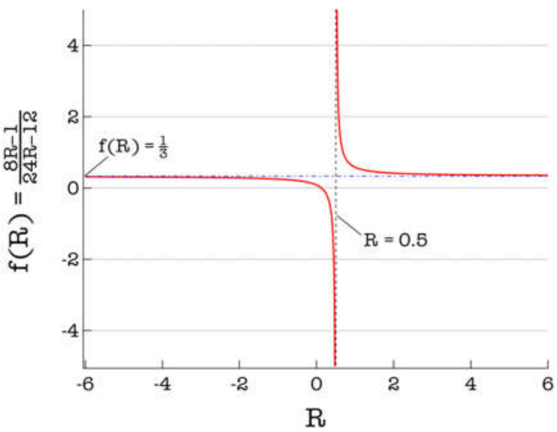

Figure 2. The nature of $f(R) = (8R-1) / (24R-12)$.

r_w [g/cm^3], g [m/s^2], and m_w [Pa · s] are water density, gravitational acceleration, and water viscosity coefficient, respectively. m_w at standard atmospheric pressure (0.1013MPa) and temperature of 20 degrees is 0.001016 Pa·s (e.g. Miyabe and Nishikawa 1968). When r_w and g are 1.0 g/cm^3 and 9.81 m/s^2, and the unit of k_w is changed to m^2 (e.g. JAGH 2010), $k_w = 0.97K$ can be obtained. Therefore, under standard atmospheric pressure and temperature, the unsaturated hydraulic conductivity k_w and the intrinsic permeability coefficient K are almost equal.

3 Verification of the proposed method in laboratory air permeability test results

Here, we verify the proposed method for specifying the intrinsic air permeability coefficient using the results of previous experiments for unsteady gas flow. This test conducted under the same boundary conditions of the exact solution as described in previous section.

3.1 Air permeability coefficient determination detection using un-steady exact solution

As shown in Fig. 3, Sato and Ono (1987) permeated nitrogen gas from the upper end of a cylindrical rock specimen and measured changes in air pressure inside the specimen. In this experiment, the pressure at the lower end of the specimen is zero $P_l = 0$, and in this study, the experimental cases shown in Table 1 are taken up as verification of the proposed method. These rocks are sampled in Japan. The Granite is from Ibaraki prefecture, which has been often used as a research sample in Japan (e.g. Oda et al., 2002; Takemura and Oda, 2005). Here, the intrinsic air permeability coefficient is obtained from the measured values in the steady state by the following equation (Sakaguchi et al., 1992).

$$K_s = \frac{2Q\mu p_a}{A}\frac{l}{P_0^2 - P_l^2} \tag{23}$$

where K_s, Q, p_a, and A are intrinsic air permeability coefficient at steady state, the amount of air permeability, air pressure, and the cross-sectional area of the specimen, respectively. Other parameters are the same as above. The experimental results are shown in Fig. 4. This shows the time series change of the air pressure measured in the center of the specimen. The results of three tests in which the value of P_0 is changed are shown. The air pressure gradually rises to a constant value. Using these unsteady data, it is verified whether the intrinsic air permeability coefficient can be appropriately specified by Eq. (21).

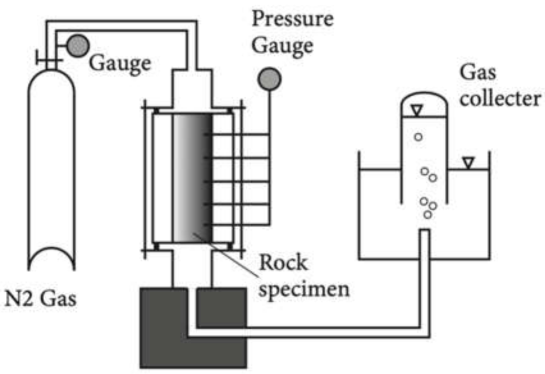

Figure 3. Laboratory air permeability test conducted by Sato and Ono (1987).

Table 1. Test cases of laboratory air permeability test by Sato and Ono (1987)

Rock type	Porosity l (%)	Specimen diameter (m)	Specimen height (m)	Case	P_0 (MPa)	K_s (m^2)
				1	0.2	1.7×10^{-16}
Granite	0.92	0.2	0.2	2	0.3	2.0×10^{-16}
				3	0.38	3.8×10^{-16}
				1	0.33	5.4×10^{-18}
Conglomerate	0.94	0.052	0.2	2	0.48	8.2×10^{-18}
				3	0.57	8.8×10^{-18}

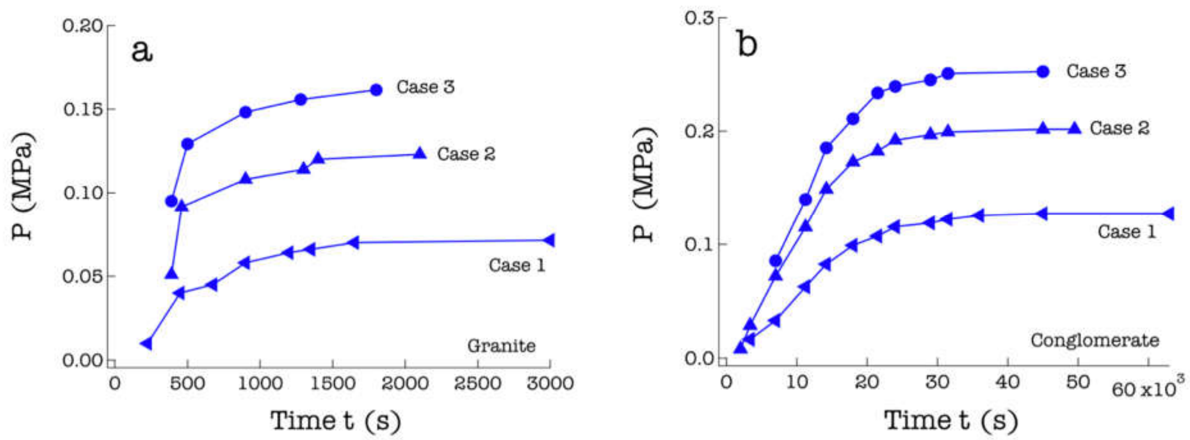

Figure 4. Unsteady gas flow of Japanese Granite (a) and Conglomerate (b) by Sato and Ono (1987): time series of P detected at the center of cylindrical specimen.

3.2 Identified intrinsic air permeability coefficient verification and its discussions

Figure 5 shows the results of determining the intrinsic air permeability coefficient K by Eq. (21) using the experimental results of unsteady pressure changes in Fig. 4. Here, the value of the air viscosity coefficient m is 1.81×10^{-11} MPa · s. As the pressure ratio increases, K increases. This is the same tendency of the air permeability coefficient K_s specified in the steady state (in Table 1), and indicates that the larger the pressure, the easier it is for air to pass through. The value of K obtained from the unsteady data using the proposed method is a little smaller than the value of K_s obtained by waiting until the steady state is reached. The magnitude of the K and K_s values is almost the same as $10^{-16} - 10^{-17}$ m^2 for granite and 10^{-18} m^2 for conglomerate. Figure. 2 shows that the method of specifying K diverges around $K = 0.5$, but if R is greater than 0.55, it is possible to specify an appropriate K without diverging.

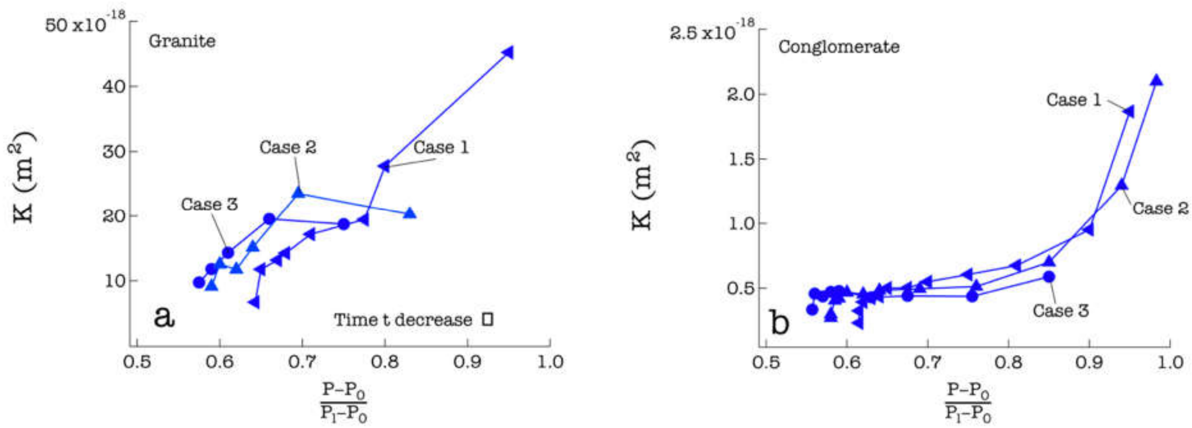

Figure 5. Relationships between identified intrinsic air permeability coefficient, K, and air pressure ratio $(P - P_0)/(P_1 - P_0)$ for Granite (a) and Conglomerate (b).

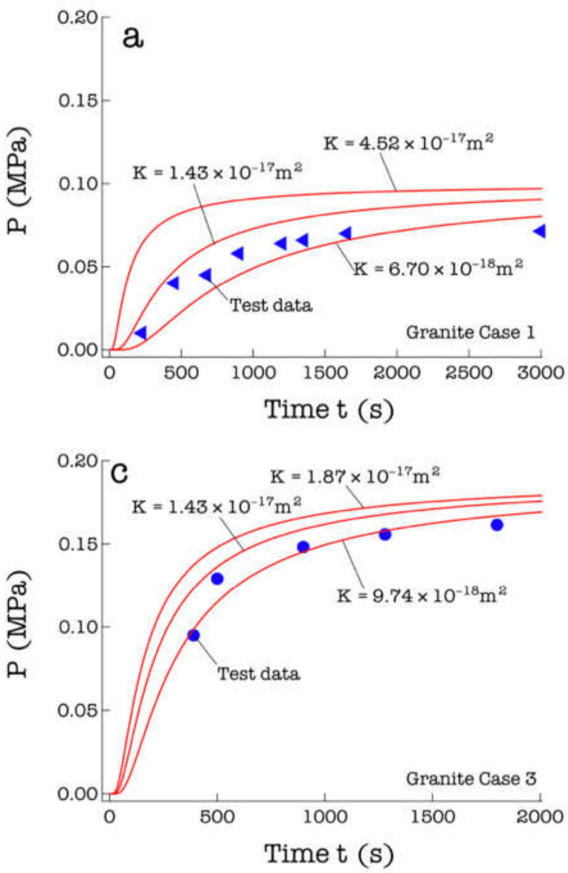

Figure 6. Comparison between experimental granite data of Fig. 4 (a) and exact solution (Eq. (14)) by using detected unsteady intrinsic air permeability coefficient, *K*.

Figure 6 compares the previous experimental results of granite in Fig. 4 with the exact solution, Eq. (14), calculated using the specified intrinsic air permeability coefficient, *K*. The figure shows an exact solution using the specified maximum, median, and minimum values of *K*. Similarly, Fig. 7 also shows a comparison of conglomerate. In all the results, the linearized unsteady exact solution using the intrinsic air permeability coefficient specified by the proposed method is in good agreement with the experimental results of Sato and Ono (1987). In particular, the exact solution using the smallest value of *K* shows a good match. This linearized exact solution assumed a small differential pressure. By this comparison, the exact solution can sufficiently express the actual phenomenon if the differential pressure $P_0 - P_1$ is about 0.57 MPa. Therefore, as stated above, the validity of both the exact solution of the linearized unsteady gas flow (Eq. (14)) and the method of specifying *K* using the unsteady data (in Eq. (21)) was shown.

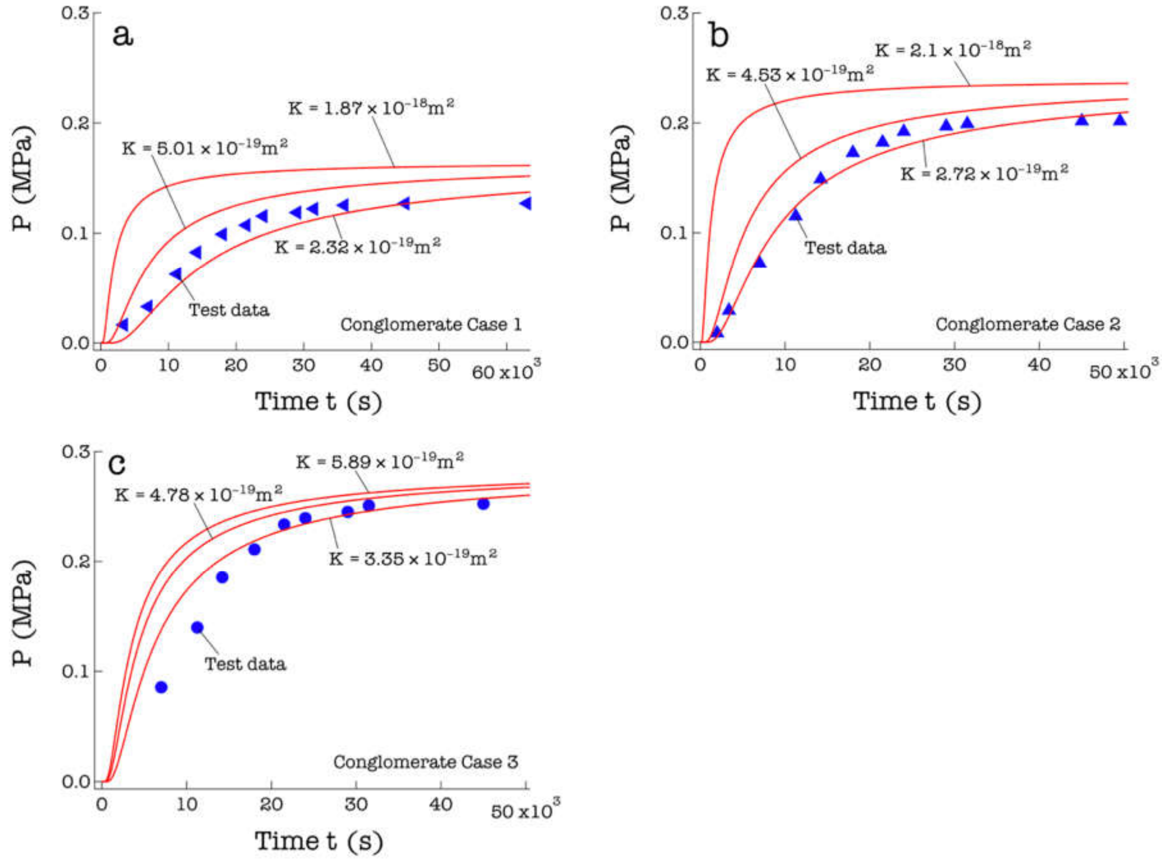

Figure 7. Comparison between experimental conglomerate data of Fig. 4 (b) and exact solution (Eq. (14)) by using detected unsteady intrinsic air permeability coefficient, *K*.

4 Conclusion

This study presents a new method to reasonably determine the unsteady air permeability using laboratory air permeability tests. Therefore, we show how to linearize the basic equation for air permeability with respect to air pressure by Katz (1959) and how to specify the air permeability explicitly from the linearized equation. The proposed method was applied using the results of previous air permeability tests to show that the identified unsteady air permeability coefficient is a reasonable value and that the linearized equations adequately represent the experimental pressure fluctuations. Future work includes further experimental investigations in the room and development of techniques to be applied to in-situ measurements.

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